Praxeological Analysis of the Mathematics Textbook on the Topic of Translation

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Abstract. This study analyzes the sequence of tasks in the Mathematics textbook to identify learning obstacles associated with the concept of translation, with a specific focus on the definition of translation. This analysis employs the theory of praxeology, a key concept in the Anthropological Theory of Didactics (ATD). The findings highlight three potential didactical obstacles and one potential epistemological obstacle within the task sequence. Therefore, these findings should be considered by teachers to anticipate and minimize potential obstacles in teaching the concept of translation.

INTRODUCTION

Transformation geometry has been introduced to Indonesian students beginning in the 9th grade, as per the 2018 revision of the 2013 curriculum. Previously, it was taught at a higher grade level. Similarly, in the latest curriculum, the Merdeka Curriculum, transformation geometry continues to be taught to 9th-grade students. The learning outcomes are categorized under Phase D (grades VII-IX), where students are expected to perform single transformations (reflection, translation, rotation, and dilation) in the Cartesian coordinate plane and apply these concepts to solve problems. This is in line with [1] standards, which state that students in grades 9-12 should be able to understand and represent translations, reflections, rotations, and dilations of objects in the plane using sketches, coordinates, vectors, function notation, and matrices, and should also be capable of utilizing various representations to understand the effects of simple transformations and their compositions.

In understanding the natural world and their surroundings, a student requires the concept of transformation geometry [2], which can aid in the development of their visual and reasoning skills [3]. Aksoy & Bayazit mention that learning the concept of transformation geometry can support students in performing analysis and synthesis, problem-solving, spatial thinking, as cited in [2], and also in conducting mathematical proofs [4]. Therefore, according to NCTM, transformation geometry is a crucial subject because mastering and subsequently applying transformation geometry enables a student to deeply explore mathematical situations.

The importance of transformation geometry, as previously mentioned and outlined in the curriculum, has not yet aligned with the ideal situation in which students should possess a correct understanding of this material. Data indicates that students still face numerous difficulties and make various errors [5]. Secondary school students continue to struggle with identifying transformations even after they have developed operational understanding, largely because most of them have not yet developed a conceptual understanding [6].

Other findings also reveal various difficulties and errors. For example, [7] found that junior high school (SMP) students have not yet mastered the use of formulas and struggle with accurate calculations in transformation topics. [8] discovered a gap between students' understanding of the concept of an image and the formal definition of transformation, where students tend to define transformation merely as a change. This understanding is inconsistent with the formal mathematical definition of transformation as a bijective mapping. The root cause of this issue is that students are introduced to the concept of transformation geometry through examples of physical changes, such as moving a table, the rotation of a propeller, or enlarging a photo, rather than through a formal definition of the transformation concept.

Furthermore, various errors and difficulties have been identified in each type of transformation geometry. [2] found that while students understand translation as a shift in position, they simultaneously struggle to determine the direction and final position of the translation. Students also face difficulties in identifying the equation of the axis of symmetry for the reflected image and in determining the angle of rotation. Similarly, [7] discovered in their research that students experience difficulties when asked to determine the translation formula given the initial and image points, identify the image of a point under reflection over the x-axis, y-axis, or the origin (0,0), and determine the image of a point after a 90° counterclockwise rotation. It was also found that the concept of rotation is not only challenging for junior high school students but also for university students.

[9] revealed that students in university often experience conceptual errors in understanding the concepts of transformation geometry. [10] similarly found that difficulties frequently arise in placing points according to definitions and in determining the image of points after transformation. Additionally, [11] identified that they struggle to prove a mapping as a transformation, including difficulties in interpreting the definition of a mapping, determining the domain and range of a mapping, and identifying contradictions in proving a mapping as an injective function. These challenges are largely attributed to students' inability to apply the fundamental concepts of geometry necessary for transformation geometry.

The difficulties and errors identified in previous studies indicate that learning obstacle have occurred in transformation geometry, particularly among junior high school students. Given these persistent difficulties, it is crucial to explore potential learning obstacles that may be inherent in the instructional design, particularly within the sequence of tasks provided in the textbook. [12] categorizes these learning obstacles into three types: ontogenic obstacles, didactical obstacles, and epistemological obstacles. When viewed from these three types of learning obstacles, several findings emerge. For instance, an ontogenic obstacle is evident where students have not yet mastered the prerequisite material required to study transformation geometry, Karso as cited in [13]. A didactical obstacle is observed when students prefer to rely on procedural knowledge rather than developing a conceptual understanding, struggle to identify related contextual problems, and are less capable of explaining the characteristics of translation, reflection, rotation, and dilation. As for epistemological obstacles, they manifest in the students' inability to apply previously learned concepts to the problems they are currently facing [13]. Additionally, [8] found in their research on didactical obstacles that the sequence of the available materials is not structurally aligned.

Referring to the findings mentioned earlier, there is a possibility that other learning obstacles have not yet been identified. Therefore, further investigation is needed to explore potential learning obstacles in transformation geometry using different theories than those previously employed by researchers. One source of learning obstacles may arise from the sequence of tasks given to students during instruction. Consequently, it is essential to analyze the sequence of tasks in the transformation geometry materials provided to students to determine whether these tasks allow students to construct new knowledge or not. If it is confirmed that learning obstacles originate from the task sequence, then the tasks in the textbooks can be revised, or teachers can prepare anticipatory measures. This study will analyze the sequence of tasks in textbooks used by students to identify potential learning obstacles. The focus will be on one aspect of transformation geometry, namely translation, particularly the definition of translation. This analysis will utilize the Praxeology theory, which is a key concept in the Anthropological Theory of Didactics (ATD), as demonstrated by [14].

METHOD

This study employs a qualitative research method, utilizing the theory of praxeology, which is a central concept in the Anthropological Theory of Didactics (ATD) [15]. ATD emphasizes the observation of human mathematical activities through the epistemological model of mathematical knowledge itself. This theory posits that knowledge should not only be viewed as objects such as concepts, ideas, or theorems but also in terms of the processes of production, use, and dissemination. Additionally, it can be used to identify students' learning obstacles. The theory then proposes modeling knowledge in terms of praxeology.

Praxeology consists of two components: Praxis (know-how), which refers to human activities, and Logos (know-why), which refers to human thought and reasoning. Each component plays a distinct role within praxeology. Praxis includes the type of task (T), which refers to the problems or situations presented as tasks to be solved, and the technique (τ), which is the method used to solve the given problem or task. Logos, on the other hand, comprises technology (θ), which is the rationale or justification for selecting the technique used, and theory (Θ), which is employed to explain and justify the technology (θ).

In this study, the students' mathematics textbooks are the documents being analyzed, with a focus on understanding the definition of translation. The textbooks analyzed include both a student book and a teacher's guide published by the Indonesian Ministry of Education and Culture.

RESULT AND DISCUSSION

A praxeological analysis was conducted on the textbooks published by the Indonesian Ministry of Education and Culture. These are the latest editions used by 11th-grade students in schools that have implemented the

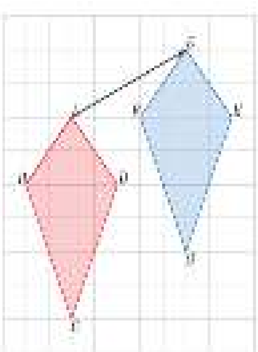
Merdeka Curriculum. Transformation geometry is covered in Chapter 3, which includes topics such as translation, reflection, rotation, congruence, and dilation. This study focuses on the topic of translation, particularly on the sequence of tasks designed to explain the definition of translation.

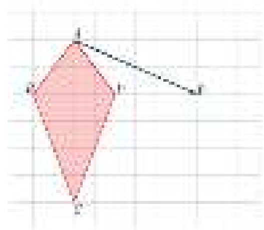
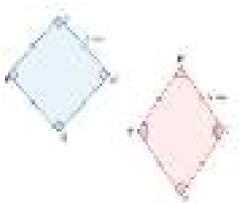
The introduction to translation begins by asking students to observe the movement of objects around them. Following this, the observation is specifically directed toward the movement of a bench, which is mentioned in the textbook as an example of translation. Therefore, in this initial introduction, students are provided with a conceptual understanding of what translation is, though without a straightforward definition.

After providing a general overview of translation, the next sequence of activities involves exploratory tasks. According to the teacher's guide, the learning objective is to explain the concept of translation. The sequence of tasks in this activity will be analyzed using praxeology.

Based on the praxeological analysis, the sequence of tasks in this exploratory activity is analyzed according to the type of task (T), which refers to the problems or situations presented as the tasks to be solved, and the technique (τ), which is the method used to solve the given problem or task. Following this, the technology (θ) is the rationale or justification for the selection of the technique used, and the theory (Θ) is employed to explain the technology (θ). The following table represents the type of task (T) aimed at explaining the definition of translation. This sequence of tasks is arranged in order from T1 to T7.

TABLE 1. Praxeological Analysis

Type of task (T)	**Technique (τ)**	**Technology (θ)**	**Theory (Θ)**
T1 : Copy the kite ABCD onto a piece of paper and then cut it out. Place the cut-out on top of the kite ABCD. Next, slide the kite cut-out along the directed line segment AE ($\overrightarrow{AE}$)  **Figure 1.** Task 1	• Trace the kite ABCD. • Cut out the traced kite ABCD. • Place the cut-out (traced ABCD) on top of the kite ABCD. • Slide the cut-out kite ABCD along the directed of line AE.	Translation is the displacement of an object in space while preserving its shape and size, according to a specified distance and direction	In geometry, translation involves changing the position of an object without altering its shape or size, referencing fundamental concepts in geometry.
T2: Is there any part of the kite EFGH that is not covered by the cut-out kite ABCD?	Examine and compare the red traced triangle and the blue triangle through in-depth visual analysis.	Analysis of form and size equivalence through in-depth visual observation.	The concept of equality and congruence in geometry, which states that two objects with the same shape and size will coincide perfectly when properly positioned.
T3: Beside $\overrightarrow{AE}$, draw the directed line segment that represents the shift of the cut-out kite ABCD to the kite EFGH	Shifting the tracing of triangle ABCD along line AE to identify other directed lines from B to F, D to H, and C to G.	Applying the principle of translation to determine the direction of each point according to the translation vector.	The theory of translation or vectors in geometry, which enables the identification of the direction of an object's movement.

Type of task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
T4: What is the relationship between side AB of the kite ABCD and side EF of the kite EFGH? How about the other sides?	Using visual observation to verify that side AB is parallel to EF, AD is parallel to EF, BC is parallel to FG, and CD is parallel to GH.	The principle of parallelism in geometry, which demonstrates that sides that are parallel before translation will remain parallel after translation.	The theory of parallelism in geometry, which asserts that translation does not alter the orientation or parallelism of a figure's sides.
T5: What are the shape and size of kites ABCD and EFGH?	Using visualization to ensure that both kites have the same shape and size.	The concept of congruence in geometry, which states that two figures with the same shape and size can completely overlap after transformations such as translation.	The principle of congruence in geometry, which refers to the similarity in shape and size between two geometric figures after transformation.
T6: Draw the kite PQRS, with its translation represented by $\overrightarrow{AP}$  **Figure 2.** Task 6	- Tracing the kite PQRS - Shifting the tracing of PQRS along line segment AP - Drawing the new triangle according to the specified direction.	Applying the principle of translation to move the object in the direction of $\overrightarrow{AP}$	
T7: Given two shapes as shown in following figure. Is the shape TUVW the result of a translation of the shape PQRS? Explain  **Figure 3.** Task 7	Using visual analysis and comparison to identify differences in size and shape between the planar figures TUVW and PQRS..	The principle in geometry that states translation preserves the shape and size of an object.	The theory of congruence in geometry, which explains that two translated figures will remain congruent if they have the same size and shape.

Subsequently, the sequence of tasks in Table 1 will be discussed, along with the suspected learning obstacles identified. Learning obstacles consist of three types: 1) Ontogenic obstacles, which are learning obstacle based on developmental readiness; 2) Didactical obstacles, which are learning obstacle caused by the incorrect selection of teaching materials and the disorganized sequence of topics; and 3) Epistemological obstacles, which are learning obstacle arising from students' limitations in understanding and applying the mathematical concepts themselves. These obstacles need to be addressed because, fundamentally, mathematical concepts are interconnected. This interconnectedness implies that the understanding of one concept will impact the understanding of related concepts (12).

The sequence of tasks (T) for understanding the definition of translation begins with T1, which involves a physical activity using tools such as scissors and graph paper provided by the teacher. Students are then instructed to trace the kite ABCD, cut out the tracing, place the cut-out on top of the kite ABCD, and slide the kite cut-out along $\overrightarrow{AE}$, as shown in Figure 1 in Table 1.

The objective of T1 is for students to discover that triangle EFGH is the result of the translation of triangle ABCD. However, in the task, both triangles ABCD and EFGH are already provided. This condition might lead to the misconception that an object and its translation already exist from the start, whereas what students should be doing is finding the result of the translation of triangle ABCD or the image of triangle ABCD. This indicates a skipped step that students should have completed before encountering the shape of triangle EFGH. This situation can be categorized as a potential didactical obstacle

Additionally, in Figure 1, there is a description stating the translation of kite A to B. This description is unclear, as A and B are points, while the text refers to the translation of kite A to B. This is likely a typographical error. Although this can be categorized as a technical error in the instructional materials, it can have significant consequences on the students' ability to acquire accurate knowledge. This can also be categorized as a potential didactical obstacle present in the textbook used.

Furthermore, in T3, students are asked to draw another directed line segment, with the answer provided in the teacher's guide as shown in the following image.

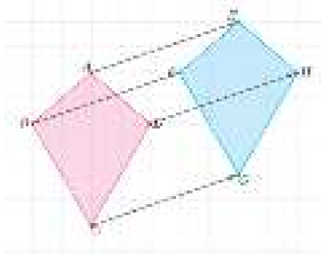


Figure 4. Answer for T3

The answer provided in the teacher's guide, if considered the only correct answer, could lead to potential misunderstandings among students, as directed line segments can only be constructed between corresponding vertices of the shape before and after translation. This could result in incomplete knowledge, leading students to believe that every point on the object is shifted using a directed line segment. Therefore, it is important to allow students the freedom to create directed line segments from any point on the kite ABCD. This situation could cause students to encounter epistemological obstacles, as they may struggle to fully understand and apply this concept.

Subsequently, in T6, students are asked to draw a kite PQRS, with its translation represented by a directed line segment. In this task, students will apply their prior knowledge of translation and directed line segments between corresponding vertices, as demonstrated in T1 and T3. The answer to this task, according to the teacher's guide, is as follows.

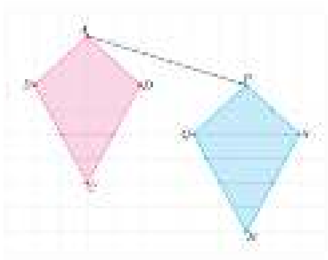


Figure 5. Answer for T6

From this answer, it appears that students are expected to immediately draw the triangle PQRS with the assistance of a single line segment $\overrightarrow{AP}$. This expectation leads to two likely possibilities: 1) students might continue to use the method from T1, tracing and then shifting according to the direction of $\overrightarrow{AP}$, and 2) students might draw additional line segments from the vertices that are parallel to $\overrightarrow{AP}$. However, if students opt for the second method, they may overlook and not yet understand a critical aspect, which is the size or length of the directed line segment.

This incomplete understanding may lead to difficulties for students in determining whether the length of the known line segment and the segment they are trying to find are the same. Even if students grasp that the lengths must be equal, how they derive the length of the directed line segment $\overrightarrow{AP}$ and then match it with the segment they are attempting to find is not facilitated in the previous sequence of tasks. This aligns with the findings of [2], who observed that while students can understand translation as a shift in position, they simultaneously struggle to determine the direction and final position of the shift. If this suspected learning obstacle occurs, it would fall under the category of didactical obstacles, as there is knowledge that was not provided in the sequence of tasks in the student's textbook.

CONCLUSION

Based on the results and discussion, it was found that the sequence of tasks provided to students to understand the definition of translation revealed three suspected didactical obstacles and one potential epistemological obstacle, particularly in T1, T6, and T3. These findings suggest that teachers should consider revising the sequence of tasks or supplementing them with additional activities that reinforce conceptual understanding. For instance, providing opportunities for students to derive the length of translation vectors through hands-on measurement or visualization exercises could address the gaps identified in T6. This approach could help to anticipate and minimize potential learning obstacles in teaching the concept of translation, thereby improving student outcomes. This study is limited to a praxeological analysis of the tasks aimed at understanding the definition of translation, suggesting that future research could extend the analysis to other task sequences within the topic of translation.

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