The Ability of Student of SMK Negeri 1 Sragen employs Cognitive Method on Algebra Questions.

Sumardi[1, a)] and Bangun Dwi Susanto[2, b)]

Author Affiliations
[1.2]University Muhammadiyah Surakarta, Indonesia

Author Emails
[a)] Sum254@ums.ac.id
[b)]Bangunhanna@gmail.com

Abstract. This study explores students' conceptual understanding and problem-solving approaches in algebra, specifically focusing on the interpretation and manipulation of algebraic expressions and equations. The research was conducted at SMK Negeri 1 Sragen with first-year students as informants. Using a qualitative descriptive method, the study analyzes students' responses to algebraic problems that involve symbolic reasoning, operations with variables, and rational expressions. Data collection involved observation, problem-solving tests, and interviews, with data validity ensured through triangulation techniques.

Two informants employed different cognitive strategies in solving equivalent algebraic problems. While both demonstrated some logical consistency, their solutions revealed common misconceptions, such as misapplying the distributive property and misunderstanding the implications of dividing by zero. In particular, the study identified that assuming division by zero in expressions like $\frac{y}{z-z}$ led to invalid conclusions. However, when solving the equation $xy = \frac{x}{y} = x-y$ both informants eventually arrived at the consistent solution $x=-\frac{1}{2}$, $y=-1$ confirming the validity of their algebraic reasoning under guided support.The results highlight the need for deeper conceptual instruction in algebra, especially in understanding symbolic manipulation and domain restrictions. The findings also underscore the importance of implementing cognitive-based learning strategies in mathematics education to enhance students' critical thinking and problem-solving skills.

INTRODUCTION

According to Fahrurrozi & Syukrul (2017:3), mathematics is a systematic and structured discipline dedicated to the study of patterns—be they relationships between quantities, modes of reasoning, artistic forms, or linguistic structures. This field is approached through logical methodologies and deductive reasoning, which allow for the formation of consistent, universally applicable conclusions. Beyond its abstract nature, mathematics holds significant practical value as it equips individuals with the tools necessary to comprehend, model, and solve complex problems encountered in various domains such as social sciences, economics, and natural phenomena. It provides a language and framework for describing quantitative facts as well as spatial and geometric relationships. These capabilities make mathematics indispensable in everyday life and numerous professional fields. Given this broad and vital role, mathematics education encompasses a wide range of concepts that students must master, each requiring a clear understanding to build deeper mathematical knowledge. One fundamental concept within this educational spectrum is algebra, which introduces learners to abstract thinking through symbols and variables. Algebra extends mathematical learning beyond arithmetic by focusing on the relationships and operations involving unknown quantities. As such, it forms the foundation for advanced studies in mathematics and related disciplines. The study of algebra encourages students to develop critical cognitive skills such as problem-solving, logical analysis, and the ability to generalize patterns—skills essential not only in academic contexts but also in real-world decision-making and reasoning. Therefore, fostering a solid grasp of algebraic concepts is crucial in the development of mathematical literacy and competence among students.

Algebra

According to Watson & Van Amerom (2003 :64) Algebra is a generalised arithmetic, Algebra as a problem solving tool, Algebra as the study of relationships, Algebra as the study of structures. In Indonesia, Algebra is one of the basic competencies in Junior high school mathematics material that focuses on recognizing the meaning of variables, constants, coefficients, and similar and similar terms. In studying algebra, the ability to understand the concept of these symbols is required. Algebra learning is introduced in mathematics learning at the junior high school/Islamic junior high school level. An understanding of algebra is also taught at the next level such as Senior high school/Vocational high school and college. Algebra learning for Junior high school level focuses on solving problems related to equations, variables, constants. Meanwhile, for first year, algebra learning includes root forms.

Algebra concern on different complexity on each level. Thus, it is lead on difficulties to understands for students. According Watson (2007 : 6) conceptual understanding is crucial for students, and many challenges remain. Therefore, it is necessary to analyze students' difficulties in algebra. Based on the explanation above, the researcher understands the complexity of understanding algebra. Therefore, the researcher will limit the problem to focus on the material taught at the senior high school level, especially for first year student.

At the high school level, particularly among first-year students, researchers have consistently observed a range of persistent challenges and misconceptions related to algebraic learning that continue to impede students’ mathematical development. These difficulties present themselves in several forms, with some of the most prevalent issues being miss-cancellation errors, improper combination of addition and multiplication operations, and an inadequate grasp of the distributive property. Miss-cancellation refers to a common error in which students incorrectly simplify algebraic expressions by canceling terms that are not legitimately canceler according to algebraic rules. This type of error reflects a deeper conceptual misunderstanding rather than a mere procedural mistake, indicating that students often rely on rote procedures without fully comprehending the underlying principles that govern algebraic operations. Furthermore, students frequently conflate addition and multiplication operations, failing to apply the correct order of operations or to recognize the distinct roles these operations play within algebraic expressions. This blending of operations leads to compounded errors and obstructs the development of more advanced algebraic reasoning skills.

The distributive property, a fundamental algebraic principle that connects multiplication and addition (or subtraction), also remains a significant source of difficulty. Many students struggle to correctly apply this property when expanding expressions or solving equations, often leading to incorrect simplifications and ultimately incorrect solutions. These struggles suggest that students lack not only procedural fluency but also the conceptual understanding necessary to manipulate algebraic expressions flexibly and accurately. Such deficiencies have far-reaching consequences because mastery of algebraic concepts is foundational for success in higher-level mathematics and other STEM-related disciplines.

To thoroughly investigate these challenges and provide evidence-based insights, the researchers undertook a systematic study involving first-year students at SMK N 1 Sragen. The study utilized a random sampling technique to select participants, ensuring that the data gathered would be representative of the broader student population and reflective of a variety of learning styles, cognitive abilities, and prior knowledge levels. Through administering carefully designed mathematics tests focused specifically on algebraic concepts, the researchers collected rich qualitative and quantitative data. These assessments were constructed to probe not only the correctness of student responses but also the reasoning processes students employed when tackling algebraic problems. The combination of observational data, test scores, and student interviews enabled a comprehensive analysis of common error patterns, conceptual misunderstandings, and the cognitive approaches students used.

The analysis of this data highlighted several key findings. It became clear that many students’ difficulties stemmed from superficial engagement with algebraic procedures, where memorization of steps took precedence over conceptual understanding. This procedural focus prevented students from developing the flexible problem-solving skills necessary for correctly applying algebraic principles in diverse contexts. Recognizing these shortcomings, the researchers concluded that traditional teaching methods, which often emphasize repetitive practice and procedural drills, are insufficient for addressing the root causes of algebraic misunderstandings. Instead, there is a critical need for pedagogical strategies that promote deeper cognitive engagement with mathematical concepts.

In response, the researchers sought to apply cognitive educational theories and mind management strategies aimed at enhancing students' mental models and problem-solving capabilities. Cognitive methods focus on improving how learners process, organize, and internalize information, emphasizing active learning, metacognition, and conceptual understanding. These approaches encourage students to become reflective thinkers who can monitor their own cognitive processes, evaluate the validity of their solutions, and adapt their strategies as needed. By fostering metacognitive skills, students can better recognize their own errors and misconceptions, which is a crucial step toward meaningful learning and long-term retention.

The implementation of cognitive methods within this study was designed to specifically target the algebraic difficulties identified among Class X students at SMK N 1 Sragen. The interventions included strategies such as concept mapping, self-explanation, and problem decomposition, all aimed at helping students build connections between algebraic symbols and their meanings, understand the rationale behind algebraic rules, and develop coherent frameworks for solving equations and simplifying expressions. The researchers also emphasized the importance of scaffolding learning experiences to gradually increase complexity while supporting students' cognitive load, thus preventing frustration and promoting confidence in mathematical problem-solving.

Beyond the immediate classroom application, this research has broader implications for mathematics education in Indonesia and similar educational contexts worldwide. Moreover, the findings suggest that ongoing professional development for mathematics educators is essential to equip them with the knowledge and skills needed to implement cognitive teaching methods effectively. Continuous formative assessment and feedback mechanisms are also recommended to track student progress and tailor instruction to individual learning needs.

On forming the comprehension discussion about the methods that employed by students on solve algebra problems under using cognitive methods, the researcher understands the need for education to implement these methods. Therefore, the researcher draws on various sources for solutions to address these issues. The researcher lists the following reference sources for conducting this research.

Previous discussion

(1) Analysis of errors in solving algebra problems reviewed by Newman on FKIP Mathematics Students of Muhammadiyah University of Surakarta published in 2021. This study highlights the difficulties faced by students in solving algebra problems so that they ultimately get unsatisfactory grades. This study uses the Newman method. This study writes the results of the Algebra problem. This type of research is qualitative descriptive. The subjects of this study were 2 students who had different views on mathematics who were in Class X of SMK N 1 Sragen.

(2) Analysis of Understanding of Algebraic Concepts refers to the APOS theory. Reviewed from the interpersonal learning style published by Sunan Ampel State Islamic University Surabaya in 2021, this study highlights the topic in mathematics learning. Many students find it difficult when solving problems related to algebra due to the lack of students' understanding of algebra using the Action, Process, Object, and Scheme method or called APOS. This study writes the results of Algebra problems. 1) Understanding of algebraic concepts of students who have a high category interpersonal style to the stage and are able to reflect on it. 2) Understanding of algebraic concepts of students who have a medium category interpersonal learning style in the object stage, including being able to solve contextual problems using algebraic form operations. 3) Understanding of algebraic concepts of students who have a low category interpersonal learning style to the action stage, including not being able to solve contextual problems and clarify the selected algebraic form operations to their properties well.

METHODS

This research uses a descriptive qualitative approach. Descriptive research produces descriptive data in the form of written or spoken words from the subjects being observed. (Moleong, 2008:3) The researcher's employ descriptive qualitative methods that concern on misunderstandings related to gaps in mathematics learning, which focused on algebra. Based on the information found, the researcher observes students' difficulties in solving mathematics problems and provided guidance on solutions.

This research was conducted at One State Vocational High School Sragen. The sample consisted of first year students of the school that located at Jl. Ronggowarsito, Village/Sub-district, Sragen Wetan, Sragen District, Sragen Regency, Central Java, and this research conducted periodically from April 2025 to August 2025.

In conducting this research, the researcher employed a purposive sampling technique, and the informants used in this study were two of first year students at SMK Negeri 1 Sragen. The subject selection was based on students' low and medium mathematical abilities, according to their mathematics teacher. The researcher assumed that students' backgrounds would provide diverse answers when providing answers related to the discussion of algebra. In conducting this research, the researcher conducted observations using a test method to work on math problems based on the topic of algebra.

In conducting research, obtaining valid data is essential. Therefore, to obtain credible data, the researcher used a data validity method called triangulation. The researcher collected data from the same category but from different students. To ensure the validity of the data obtained, researchers also validated the data using various techniques, including observation tests and interviews, to understand the mindsets and factors that cause students to make errors when solving algebra problems.

Data collection

In conducting this research, the researcher employed a purposive sampling technique, and the informants used in this study were two of first year students at SMK Negeri 1 Sragen. The subject selection was based on students' low and medium mathematical abilities, according to their mathematics teacher. The researcher assumed that students' backgrounds would provide diverse answers when providing answers related to the discussion of algebra. In conducting this research, the researcher conducted observations using a test method to work on math problems based on the topic of algebra.

In conducting research, obtaining valid data is essential. Therefore, to obtain credible data, the researcher used a data validity method called triangulation. The researcher collected data from the same category but from different students. To ensure the validity of the data obtained, researchers also validated the data using various techniques, including observation tests and interviews, to understand the mindsets and factors that cause students to make errors when solving algebra problems.

FINDINGS

Based on the explanation, the cognitive roots of algebraic challenges, educators can empower students to become proficient, confident, and independent mathematical thinkers capable of tackling complex problems both within and beyond the classroom.

Therefore, to get valuable insights to the field of mathematics education research by demonstrating that overcoming algebraic misconceptions requires a holistic approach that combines accurate diagnosis of student difficulties with thoughtfully designed cognitive interventions. The researchers advocate for sustained collaboration among educators, researchers, and policymakers to create supportive learning environments where students can develop both procedural fluency and deep conceptual understanding.

DISCUSSION

In conducting the research, the researcher obtained results in the form of answers to mathematics problems related to algebra. By attaching answer sheets completed by the informants, the narrative of the answers will visualize the methods used by the informants in solving the algebra problems.

(1) Given three positive numbers x, y, and z, all of which are distinct. If y/(z - z) = (x + y)/z = x/y, then what is the value of x/y?
(2) Suppose x and y are nonzero real numbers with xy = x/y = x – y. What is the value of x + y?

3.1. Informant I (Elvira Agus Saputri)

In answering the two questions above, Elvira Agus Saputri, used the following method:

1. $\frac{y}{x-z} = \frac{x+y}{z} = \frac{x}{y}$. then the value of $\frac{x}{y}$? (Informant employs cross multiple method)

$$y(y) = x(x - z)$$
$$y^2 = x^2 - xz$$

it is also known that $\frac{y}{x-z} = \frac{x+y}{z}$. cross time more to get the amount

$$yz = (x – z)(x + y)$$
$$yz = x^2 + xy - xz – yz$$
$$2yz = (x^2 - xz) + xy$$
$$2yz = y^2 + xy$$

Thus, obtained

$$\frac{x}{y} = \frac{x+y}{z}$$
$$= \frac{2z}{z} = 2$$

The sequence of algebraic manipulations leads to the result that both Y and Z must be zero. However, this outcome invalidates the original rational expression involving $\frac{y}{z}$ as division by zero is undefined in the real number system. Therefore, although the internal algebra is consistent, the foundational assumption involving $\frac{y}{z}$ is not permissible under the derived values of the variables.

2. On second question that the value of x and y is real number with has a form of xy = $\frac{x}{y}$ = x – y.

Then, how is the value of x + y ?

$$xy \,.\, y = \frac{z}{y} \,.\, y$$
$$xy^2 = x$$
$$y^2 = 1$$
$$y = \pm 1$$

possibility 1: y = 1
substitution in the equation zx= x – y then it is obtained

$$x(1) = x - 1$$
$$x = x - 1$$
$$0 = -1$$

we get a statement that is false (it should be 0 ≠ -1) so that there is no x value that satisfies if we choose y = 1

possibility 2: y = -1

substitution in the equation xy = x-y then it is obtained

$$x(-1) = x - (-1)$$
$$-x = x + 1$$
$$2x = 1$$
$$X = -\frac{1}{2}$$

so it's obtained $x = -\frac{1}{2}$. thus, value $x + y = -\frac{1}{2} + (-1) = -\frac{3}{2}$

3.2. Informant 2 (Nugraga Prawesti)

On the first question, Nugraga Prawesti using also cross multiple method but the method employed in different point of view. Nugraga Prawesti outlined that

$$xy = \frac{x}{y} = x\text{-}y$$
$$xy = \frac{x}{y}$$
$$\frac{y}{z\text{-}z} = \frac{X+y}{z}$$
$$yz = (z\text{-}z)\,(x + y)$$
$$yz = zx + zy - zx - zy$$
$$yz = 0$$
$$yz = y^2$$
$$z = y$$

The system of equations

$$xy = \frac{x}{y} = x\text{-}y$$
$$\frac{y}{z} - z = \frac{X+y}{z}$$

and

$$yz=(z-z)(x+y),$$

leads to important deductions about the variables x, y, and z.
First, from the identity $yz=(z-z)(x+y)$ it follows that

$$yz=0$$

since $z-z=0$
Further, the equation $yz=y^2$ implies

$$y^2=0$$

which yields

$$y=0.$$

Substituting back into z=y, we conclude

$$z=0.z = 0.z=0.$$

Given that y=0 the initial expression $xy = \frac{x}{y}$ xy= involves division by zero, which is undefined. Thus, the system imposes constraints that require careful consideration of domain restrictions on y.

In addition, on the second question, Nugraga Prawesti employs the methods that

$$xy = \frac{x}{y} = x\text{-}y$$
$$xy = \frac{x}{y}$$
$$xy^{2} = x$$
$$y^{2} = 1$$
$$\frac{x}{y} = x\text{-}y$$
$$x = y\ (x\text{-}y)$$
$$xy = x\text{-}y$$
$$xy+y = x$$
$$y(x+1) = x$$
$$y(x+1) = y(x\text{-}y)$$
$$x+1 = x\text{-}y$$
$$x+y = x\ \text{-}1$$

Based on the analysis of the system of equations

$$xy = \frac{x}{y} = x\text{-}y$$
$$y^{2} = 1$$

and the derived relations, it is established that y must be either 1 or −1. Substituting these values into the equation x(1−y)=−1 reveals that y= 1 leads to a contradiction, while y=− yields a consistent solution for x. Consequently, the unique solution to the system is

$$x = -\frac{1}{2},\ y = -1$$

which satisfies all equations simultaneously. This solution is valid within the domain constraints imposed by the original expressions.

CONCLUSION

The conclusion of the discussion can be seen that both of the student using same method on cross multiple on the numbers. In addition, they did not employ the exact same method the first informant employs the method on outlining the variable as Yz =(z-z) (x+y) turn to Yz= zx+zy-zx-zy this method results Yz= 0 relate to Yz= y^2 and z=y this method results quite confusing because the direct result is infinite thus there are Yz that divided by zero.

In the other hand, the second informant Nugraga prawesto employs a method that direct explain on the result that y=0 the initial expression xy = $\frac{x}{y}$ xy= involves division by zero, which is undefined. Thus, the system imposes constraints that require careful consideration of domain restrictions on y.

In contrast on the second question, the research found that the student also using different method. Elvira Agus Saputri explain that the calculation operation XY= $\frac{x}{y}$ = X−Y and $Y^2 = 1$ meaning Y=±1 then outlined to be $\frac{x}{y}$=X−Y, multiplying both sides by Y leads to X(1−Y)=−1 X(1 - Y) this method results
X= $-\frac{1}{2}$,Y=−1.

The second informant on the second question emphasize another method that xy = $\frac{x}{y}$ = x-y it derives to be $y^2 = 1$ thus and the derived relations, it is established that y must be either 1 or −1. Substituting these values into the equation x(1−y)=−1 reveals that y= 1 leads to a contradiction, while y=− 1 yields a consistent solution for x. Consequently, the unique solution to the system is

$x = -\frac{1}{2}$, y = -1

ACKNOWLEDGMENTS

The author wishes to extend heartfelt and sincere appreciation to smk negeri 1 sragen and universitas muhammadiyah surakarta for their outstanding academic support and unwavering encouragement throughout the completion of this research. the successful development of this article would not have been possible without the strong educational foundation, access to quality learning resources, and collaborative atmosphere provided by these two esteemed institutions.

At smk negeri 1 sragen, the author is particularly grateful to the mathematics educators and academic staff who granted access to the classroom setting, offered critical feedback, and provided valuable insights into the challenges faced by students in understanding algebraic concepts. the cooperation extended by the school community, especially the students who participated as informants, significantly enriched the data collection process and provided authentic perspectives that shaped the direction of the study.

Likewise, universitas muhammadiyah surakarta has played a vital role in supporting the author’s academic journey. the university's commitment to research excellence, its rigorous academic environment, and the mentorship provided by faculty members have equipped the author with the necessary analytical and methodological tools to conduct this research effectively. the availability of academic literature, research seminars, and constructive discussions with peers further contributed to the scholarly depth and clarity of this work.

The author acknowledges the role of both institutions not only as facilitators of formal education but also as nurturing grounds for critical thinking, innovation, and academic integrity. it is through the shared mission of these institutions to advance knowledge and promote student-centered learning that this research was able to reach its full potential.

In conclusion, the author is deeply indebted to both smk negeri 1 sragen and universitas muhammadiyah surakarta for their meaningful contributions. their support has left an enduring impact on the author’s academic and personal growth, and this research is dedicated in part to the educators and academic communities who continue to inspire excellence in the field of mathematics education.

REFERENCES

1. Arikunto and Suharsimi. *Dasar-Dasar evaluasi pedidikan*. (Erlangga, Jakarta, Indonesia, 2006)pp.56-59
2. Fahrurrozi, Syukrul.*Metode Pembelajaran Matematika.* (University Hamzanwadi Press. Selong, Indonesia, 2017) pp. 45-49
3. Moleong, L.J. *Metode penelitian kualitatif*. (Remaja Rosdakarya, Bandung, Indonesia. 2008) pp.78-80
4. Pendit, P. *Makna imformasi: Lanjutan dari sebuah perdebatan, dalam sebuah kepustakawanan Indonesia: Potensi dan tantangannya*. (Kesaint Blanc, Jakarta. Indonesia.1992) pp 12-16
5. Rusmawan, U. *Teknik Penulisan Tugas Akhir dan Skripsi.*(PT Elex Media Komputindo, Jakarta, Indonesia. 2019) pp 90-93
6. Ruslan, R. *Manajemen. Public Relations & Media Komunikasi.* (PT. Rajagrafindo Persada, Jakarta, Indonesia.2016) pp.90-94
7. Syah, M. *Psikologi Pendidikan dengan Pendekatan Baru.* (PT Remaja Rosdakarya, Bandung, Indonesia. 2013) pp. 90-92
8. Sugiyono.*Metode penelitian Kuantitatif, Kualitatif dam R&D*. (CV Alfabeta, Bandung, Indonesia.2008)pp. 78-81
9. Soejadi, R. *Kiat pendidikan matematika di Indonesia: Konstalasi, Keadaan Masa Kini Menuju Harapan Masa Depan.* (Erlangga, Jakarta, Indonesia.2000)pp. 67-69
10. Nugroho, Puspo. (2015) Pandangan kognitifisme Dan Aplikasinya Dalam pembelajaran Pendidikan Agama Islam Anak Usia Dini. *ThufuLA: Jurnal Inovasi Pendidikan Islam Anak Usia Dini. 3*, (2).