

# Hypothetical Learning Trajectory (HLT) to Build Understanding of Mathematics Education Students about What is and How to Apply Problem Based Learning (PBL) to Learn Mathematics

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**Abstract.** The study aims to (1) find out how Hypothetical Learning Trajectory (HLT) is in problem-solving learning to study the volume and surface area of a cube, (2) find out how the understanding of S1 students who take class D Micro Teaching courses regarding Problem Base Learning models is. The type of this research is design research. The subjects in this study were 15 students who took Micro Teaching courses in class D. The data analysis used in this study, used data analysis steps formulated by Miles and Huberman, namely data reduction, data presentation, drawing conclusion, and verify. In general, the learning steps taken are as follows (1) the researcher conveys the learning objectives, namely helping students understand problem based learning, (2) the researcher gives apperception to the topic used as a problem-solving medium, (3) the researcher conveys the problem to solve the problem completed by students, (4) students solve the problems given and are accompanied by researchers then present them, and (5) evaluate the result of problem solving then students draw conclusions from a series of activities carried out during teaching. Based on the results of interviews after going through the learning process, it can be concluded that eleven students can only explain that PBL must begin with solving problems that aim to find a mathematical concept. Then four students can define how the stages of learning that must be passed by students, starting from giving problems to finding a mathematical concept.

## INTRODUCTION

Mathematics is one of the subjects that must be studied at all levels of education, starting from basic education to higher education. Mawarsari (2014) argues that efforts to improve the quality of Mathematics teachers as resources that play a role in the Mathematics learning process is to improve the ability of Mathematics education teachers and students as prospective Mathematics teachers. Suggested by Shulman (in Helen Chick, Monica Baker, Thuy Pham, & Hui Cheng, 2006) that the basis for teaching is content knowledge, pedagogy, curriculum, students, educational context and educational goals. Shulman (1987) also mentions that Pedagogic Content Knowledge (PCK) describe a mixture of learning content and pedagogy which is the teacher's domain by involving "an understanding of how certain topics, problems or issues are organized, represented and adapted to the diverse interests and abilities of students and served for teaching" (Helen Chick, Monica Baker, Thuy Pham, & Hui Cheng, 2006).

Profound Understanding of Fundamental Mathematics (PUFM) is described by Ma (1999, in Helen Chick, Monica Baker, Thuy Pham, & Hui Cheng, 2006) as a basic understanding of Mathematics with

depth, breadth and accuracy. Teachers with this understanding make connections between concepts and procedures approach concepts and problems from multiple perspectives showing an explicit awareness of “simple but powerful basic concepts”, and have an understanding of the curriculum as a whole rather than knowledge of the parts to be taught. The National Council of Teachers of Mathematics (NCTM, 1998) formulates the ability to learn Mathematics which is called mathematical power, which includes: (a) mathematical communication, (b) mathematical reasoning, (c) mathematical problem solving, (d) mathematical connection, (e) learn to represent. Therefore, if you want to be a good Mathematics teacher, then the teacher must understand, have these five competencies, and know what processes need to be carried out learning Mathematics to develop these five abilities in students.

Mathematical Content Knowledge (MCK) was formulated by Ball etc. (Goos, 2013), (1) common content knowledge, (2) specialized content knowledge, (3) knowledge at the mathematical horizon. MCK and PCK is a combination that needs to be owned by prospective teachers or Mathematics teachers, so that students have good achievements in the field of Mathematics. In research conducted by Merrillyn Goos (2013) PCK has a greater influence than MCK. But PCK cannot be done without MCK. According to the research result of Sofi Nurqolbiah (2016) and Asfi Yuhani, et al (2018) the results show that students who are taught using Problem Based Learning (PBL) show a positive attitude in dealing with problems and their problem-solving abilities are superior to students who are taught using ordinary learning or a scientific approach. Christina Collet and Regina Brother (2006) concluded that with the intervention in the field, problem solving learning leads to an increase in the effect of student learning in students' mathematical performance, especially in problem solving in the competencies taught.

Therefore, Mathematical Education students who are prospective Mathematics teachers need to be prepared in carrying out Mathematics learning using PBL. One of the efforts that can be done to prepare these prospective teachers is to introduce what PBL is and how the process of doing Mathematics learning using PBL in Micro Teaching courses is. In the Micro Teaching learning process in Class D, in one practical opportunity, students were asked to teach one of the Mathematics materials in Junior High School, using PBL. From the practice carried out by students, researchers obtained the following problems: (1) when students had difficulty solving problems, the teacher explained how to solve the problem and the teacher did not provide guidance to students, (2) the teacher explained the concepts that would be built by students through the problem solving process, so that from the process of solving the problem into the process of implementing knowledge, which should be built through the problem, and (3) the teacher explains the knowledge that should be built by students through the problem, after that the teacher does not give a problem but immediately gives practice questions to implement what explained by the teacher.

The existence of problem in the practice of Micro Teaching Class D as described above, makes researchers plan to develop a learning trajectory that can help students understand what PBL is and how to teach Mathematics using PBL. According to rezky (2019) the learning trajectory must describe how Mathematics educators (teachers, researchers, and curriculum developers) are oriented towards a constructive process, how mathematics teachers help students to design problem solving strategies and implement these strategies so that students can build the expected mathematical concept. Simon (in Nurdin, 2011) argues that a learning trajectory must include three main components, namely learning objectives, a set of tasks to achieve these goals, and hypotheses about how students learn and how students think. In the learning trajectory that will be built by the researcher, the researcher will use the volume and surface area of the cube as a vehicle to bring students to understand what PBL is and how teach Mathematics using PBL. PBL have five stages, namely (1) introducing students to problems; the educator conveys the learning objectives to be achieved, checks students' perceptions by asking questions about the previous material, and provides motivation, (2) organizes students to learn, (3) assisting independent and group investigation; educators provide support and encourage students to collect data and conduct experiments independently or in groups, (4) develop and present their work, and (5) analyze and evaluate the problem solving process (Arends in Hilyatin and Qohar, 2015)

Based on the existing problems, it can be concluded that the formulation of the problem in this research is (1) how to make a learning trajectory to help students understand what PBL is and how to teach volume and surface area of a cube using PBL?, and (2) how to understand S1 Students of Mathematics Education at Sanata Dharma University who are taking Micro Teaching courses in class D about what PBL is and how to teach the volume and surface area of a cube using PBL?

## RESEARCH METHODS

This research using design research. Design research is appropriate to be used in this study because in this study the researcher builds a learning trajectory that helps students to understand what PBL is and how to teach the surface area and volume cube material using PBL, and the researcher looks at the impact of the learning trajectory made by the researcher on the understanding of the students about what PBL is and how to teach surface area and volume cube material using PBL. The subject in this study were 15 students of the Mathematics Education study Program at Sanata Dharma University who took the Micro Teaching course in class D. The data in the study were obtained from questionnaires distributed after learning was carried out and interviews with research subjects related to understanding what PBL is and how to teach the surface area and volume of a cube material using PBL. The process of analyzing research data is carried out according to the stages of data analysis according to Miles and Huberman (1992), namely (1) data reduction, (2) data presentation, and (3) drawing conclusions/ verification.

## RESULTS AND DISCUSSION

The learning process of the research class lasted in one session for 2 x 50 minutes (100 minutes). The aim of this learning is to help the university student understanding what the teaching and learning process is and how to teach about the formula of cube's surface large and its volume through the teaching and learning process. The subject of this research class is 18 students, and it has been held on 10 May 2021. On this learning there were two cases concerning with solid cube. They are as follows:

### Case 1

There's a solid cube, its side is 4 cm. That cube will be painted in terms of these:

- a. There were 3 different colors to paint it, but it's not allowed to paint 3 sides with the same color.
- b. Every side of the cube is painted with the same color.
- c. After being painted and dried, the solid cube was cut into smaller cubes in 1 cm long.

The questions:

- a. How many small cubes which set the solid cube?
- b. How many small cubes which were painted with a color, 2 colors, 3 colors? How many small cubes which was painted.

### Case 2

Based on the first case

- a. How many small cubes which have 2 sides colored?
- b. How many small cube which have 2 sides colored?
- c. How many small cubes which has a side colored?

The process which happened in the class based on the cross learning which has been arranged by the researcher as follows:

- 1) Phase 1: Giving such orientation about the case to the student's
  - a) The teachers started the learning by reviewing the student's readiness and sharing the aim of the learning which are going to do that is the students will construct the case relates to the cube to know the learning process through problem based learning.
  - b) The teachers invited the students to recall the characteristics of the cube. The researcher provided a picture of a cube in the form of a cube frame on a slide and then asks verbally "Do you still remember, what are the characteristics of the cube?"
  - c) Some students tried to answer that the cube has 6 sides that are square, the cube has 12 edges which each side being the same length, the cube has 8 corner.

- d) The teachers invited the students to know solid cubes by displaying a picture of a cube that is different from the first cube. Then the teacher will ask the students verbally "What is the difference between the first and the second cube?"
- e) Some students tried to answer spontaneously, "The picture of the first cube is just a cube frame while the second cube looks solid or all space are filled and no gaps".
- f) The teachers gave problems related to the cube
- 2) Phase 2 : Organizing the students
  - a) The teachers asked the students what have the students understand about the problems which were presented and were there any misunderstandings experienced by students about the problems presented. Then the teachers gave the opportunity to the students to find initial ideas in constructing the problems
- 3) Phase 3 : Guiding individual or group investigations
  - a) After the teachers knew clarification from students regarding of students' understanding on problem, the teachers asked the initial idea that students will do to solve the problem.
  - b) The students explained the selected coloring pattern, "I used first color for 2 sides, second color for 2 sides and third color for 2 sides cube."; "I used first color for 3 sides, second color for 2 sides and third color for 1 side cube."
  - c) The teachers gave a time for the students to solve the problems which have been presented by occasionally asking whether there were difficulties experienced by students, if there is no answer the teachers will call several name of the students to ascertain whether there are difficulties or not.
- 4) Phase 4 : Presenting the result
  - a) The teachers asked the students to compile the results that have been obtained.
  - b) The teachers asked the students several times to make sure whether the students have solved the problem and compiled it or not.
  - c) The teachers asked the students in order to know who have finished and asked then to send the answers to the class chat group.
  - d) When the time is up, the teachers gave the opportunity to students who are willing to present the result.
  - e) The teachers provided clarification on the answers obtained by the students.
  - f) The teachers asked the students if there were different answers, some students got different answer and not correct result, then the teachers provided support to guide students to get the right result.
- 5) Phase 5 : Analyzing and Evaluating the problem-solving process
  - a) At this phase, the teachers helped the students to reflect and evaluate the learning process and results of the investigations which students have done by asking this question: "What did you find from today's learning process?". The following are the result of sharing from five students:
    - i. "We reviewed the characterize of the cube and what a solid cube was", "We should solve the cube's problems where the solid cube was painted, cut and found how many small cubes are there and how many sides of the small cube are painted".
    - ii. "We mentioned volume and surface area from the problem has given, where the volume described from the number of small cubes and surface area from the total number of all painted cube's sides."
    - iii. "Today's learning has used problem-based learning, because in the previous learning it was started with a problem and then the problem will be solved by students so that they can discuss a certain achievement."
    - iv. "Today's learning has used the PBL approach, we were given a problem before then we experimented to solve the problem."

- v. "Today's learning has implemented PBL, because at the beginning we were given a problem, then we were directed to do what it was like, then we worked on and solved the problem after that we also presented the results of our answers and at the end we did reflection on learning."
- b) Furthermore, the teacher also gives reflection questions in the form of google form to students. The questions on the google form are as follows: "In your opinion, what is PBL, and how to teach Mathematics using PBL?"

#### *Description of Students Learning Results in Research Classes*

The research was carried out on 6th semester students in the Class D Micro Teaching course, a total of 18 students with different abilities. Students are given two problems to study as a support in understanding problem based learning. Researchers analyzed the test results based on the results of students work using the research data analysis process according to Miles and Huberman, as follow:

##### *(a) Students Learning Results on First Problem.*

Based on the collection of the results of problem solving 1, 11 of 18 students have collected the results of the solution and there are three groups of answers based on the solution of the given problem, namely (1) the first group of answers there are nine students with the same answer, in this group of answer, the cube is painted with the pattern 2,2,2. As well as mentioning that the number of small cubes that make up a large cube is 64 small cubes, and the number of sides of a small cube that is painted is 96 sides of a small cube. Or it can be interpreted that the surface area of the cube is  $96 \text{ cm}^2$ . (2) the second answer group consisted of two students with a pattern of painting done was 3,2,1. As well as mentioning that a cube that is partitioned into small cubes with many small cubes is 64 pieces or calculated by  $4 \times 4 \times 4 = 64$  which represents the volume of a cube with a side length 4 cm is  $64 \text{ cm}^3$  and the number of sides of a small cube that is exposed to paint is  $48 + 32 + 16 = 96$  sides. (3) the third answer group, there are two students who have not been able to find a solution to the problem presented.

For the first and second answer groups, the researcher can conclude that both groups have successfully solved the problem and obtained the right results, the two groups of answers have fulfilled the five mathematical content knowledge according to (NCTM, 1998) namely (1) mathematical communication, at this stage students communicate the problems presented to teachers, (2) mathematical reasoning, at this stage this group of students find initial ideas in solving problems, (3) mathematical problem solving, at this stage the students group has used various strategies to solve problems, (4) mathematical connection, at this stage the students group has linked their knowledge with ideas obtained to solve the problem, and (5) learning to be representative, at this stage the students group is able to represent mathematical ideas in solving problems.

The third group of answers is a group of students who have not been able to solve the problems given. This group of students has not fully fulfilled by students in the third group are, (1) mathematical communication. Students have also tried to communicate their understanding of the problems, and (2) mathematical reasoning, at this stage students have tried to convey the initial ideas they found to solve the problem. This group of students have not been able to relate the initial

Ideas they found with their ability to solve problems, so this group of students have not been able to solve the problems.

##### *(b) Students Learning Results on the Second Problem.*

For the second problem, 11 out of 18 students have collected the results of the solution and there are three groups of answers based on the solution of the given problem, namely (1) the first group of answers is the group of students with the correct completion steps, (2) the second group of answers is the group. They are less thorough in analyzing the problems given, (3) the third group is students who have not been able to solve the problem and given.

Based on students learning outcomes in first problem and second problem, it can be concluded that students in the first and second answer groups for first problem and students in the first group for second problem meet the indicators of MCK formulated by Ball etc. (Goos, 2013) in solving problems about the surface area and volume of a cube, where students have; (1) common content knowledge, where students are in the field of mathematics, (2) specialized content

knowledge, where students understand the characteristics, surface area and volume cubes, and (3) knowledge at the mathematical Horizon.

*Interview Result : The Understanding Problem Based Learning*

Because of there were some student who did not fill out the Google Form, the researchers conducted an interview by Whatsapp through the students. Based on the results of interviewed 15 students with the question "In your opinion, what is PBL, and how to teach Mathematics using PBL?", we found that there are three groups of answers. The following is an excerpt from an interview from each group:

1) The First Group's answers:

The following is an excerpt from an interview with one of the five students in the first group of answers:

Q : "In your opinion, what is PBL, and how do you teach Mathematics using PBL?"

M1: "Problem-based learning model is a learning model that at the beginning of learning we start with contextual problems (daily life) and the teacher expect that students can solve the problems by using their thinking skills so that students can be brought to the formation of mathematical concepts".

2) Second group's answers:

The following is an excerpt from an interview with one of the six students in the first group of answers:

Q : "In your opinion, what is PBL, and how do you teach Mathematics using PBL?"

M2: "As far as I know PBL is learning that emphasizes the process of forming mathematical concepts that begins with giving students problems to solve and through this process students are expected to be able to think critically so they can find these concepts"

3) Third group's answers:

The following is an excerpt from an interview with one of the four students in the first group of answers:

Q : "In your opinion, what is PBL, and how do you teach Mathematics using PBL?"

M3: "According to what I understand about PBL based on lessons learned on May 10: (1) students are given a problem related to the topic of the cube. (2) students are helped to solve the problem, with the help and guidance of the teacher to get information from the questions given. (3) students are then asked to convey the results of solving the problems given; (4) students and teacher made conclusions from the learning that has been done."

Based on the explanation above, we can conclude that (1) five students said that PBL is learning that begins with contextual problems which students are then brought to the process of finding mathematical concepts. These five students in this group have not demonstrated PCK ability, because they do not explain how the process or steps that must be taken by both students and educators starting from the process of giving problems to finding concepts; (2) the second group's answers (six students) said that PBL is learning that begins with presenting problems that are designed in a context that is relevant to the material being studied, and makes students able to think critically so that they can find a concept. These six students did not explain how the process or steps that must be taken by both students and educators starting from the process of giving problems to finding concepts and do not explain why the process of giving problems can make students able to think critically. This shows that the six students in this group have met the characteristics of PCK according to Shulman (1987) that Pedagogic Content Knowledge (PCK) describe a mixture of learning content and pedagogy which is the teacher's domain by involving "Understanding of how certain topics, problems or issues are organized, represented and adapted to the interests and abilities of diverse learners and presented for teaching" (Helen Chick, Monica Baker, Thuy Pham, & Hui Chng, 2006). Students understand the learning that is being carried out, namely to understand what PBL is and knowledge about PBL and be able to represent; and (3) the third group's answers (four students) said that PBL is a learning process with the following steps: (a) students are given a problem related to the mathematics material to be discussed; (b) students are guided by the teacher to obtain information from the questions were given, so that students can solve the problem; (c) students are asked to convey the results of solving the problems given; and (d) students and the teachers make conclusions from the learning that has been done. The four students have explained



the steps experienced by students in the learning process using the PBL learning model, but have not explained that through this process students are invited to carry out the process of constructing mathematical knowledge related to the problems solved by this study. The four students in this answer group have done content knowledge, but have not fulfilled the pedagogical knowledge. Where students know what topics are being studied but have not been able to represent what PBL is.

## CONCLUSION

Based on the research that has been done, it can be concluded as follows:

1. The steps to teach Mathematics using PBL to find the volume and surface area of a cube are as follow;
  - a. The teachers explained the learning objectives that will be carried out.
  - b. The teachers gave apperception so that students get an idea related to solving the next problem.
  - c. The teachers gave two problems about the process of painting solid cube using three colors for students to solve and the teacher provides support for students.
  - d. Students presented the result of problem solving they get, then the teacher evaluate the result of broblem solving obtained by students.
  - e. At the end of the lerning the teacher and students were doing the reflection of what have been experienced during the learning process, and conclude that the way to find the volume of a cube is to multiply the lenght of the side of the cube by the lenght of the side of the cube and multiply it again by the lenght of the side of the cube and how to find outer surface of the cube is six time the surface area of one side of the cube.
2. From the results of interviews with fifteen students, the following results were obtained:
  - a. Five students did not explain how the process or steps that must be taken by both students and teacher starting from the process of giving problems to finding concepts.
  - b. Six students did not explain how the process or steps that must be taken by both students and teachers starting from the process of giving problems could make students able to critical thinking.
  - c. Four students have explained the steps experienced by students in the learning process using the PBL , but have not explained that through this process students ar invited to carry out the process of constructing Mathematical knowledge related to the problems solved by these students.

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